

Solución ejercicios repaso para examen de Evaluación. ACT II

1.

a) $(3x^3 - 4x^2 - 9x + 17) : (x - 2)$

$$\begin{array}{r|rrrr} & 3 & -4 & -9 & 17 \\ 2 & & 6 & 4 & -10 \\ \hline & 3 & 2 & -5 & 7 \end{array}$$

Cociente: $3x^2 + 2x - 5$
Resto: 7

b) $(3x^5 + 2) : (x + 1)$

$$\begin{array}{r|rrrrrr} & 3 & 0 & 0 & 0 & 0 & 2 \\ 1 & & 3 & 3 & 3 & 3 & 3 \\ \hline & 3 & 3 & 3 & 3 & 3 & 5 \end{array}$$

Cociente: $3x^4 + 3x^3 + 3x^2 + 3x + 3$
Resto: 5

c) $(x^5 + 4x^2) : (x + 3)$

$$\begin{array}{r|rrrrrr} & 1 & 0 & 0 & 4 & 0 & 0 \\ -3 & & -3 & 9 & -27 & 69 & -207 \\ \hline & 1 & -3 & 9 & -23 & 69 & -207 \end{array}$$

Cociente: $x^4 - 3x^3 + 9x^2 - 23x + 69$
Resto: -207

d) $(2x^4 + 3x^2 - 5) : (x + 4)$

$$\begin{array}{r|rrrrr} & 2 & 0 & 3 & 0 & -5 \\ -4 & & -8 & 32 & -140 & 560 \\ \hline & 2 & -8 & 35 & -140 & 555 \end{array}$$

Cociente: $2x^3 - 8x^2 + 35x - 140$
Resto: 555

e) $(2x^5 - 3x^4 + 4x^3 - 5x^2 + 3x + 1) : (x + 2)$

$$\begin{array}{r|rrrrrr} & 2 & -3 & 4 & -5 & 3 & 1 \\ -2 & & -4 & 14 & -36 & 82 & -170 \\ \hline & 2 & -7 & 18 & -41 & 85 & -169 \end{array}$$

Cociente: $2x^4 - 7x^3 + 18x^2 - 41x + 85$
Resto: -169

② a) $x^4 + 3x^3 - 3x^2 - 11x - 6 = (x-2)(x+3)(x+1)(x+1) = (x-2)(x+3)(x+1)^2$

| | | | | | |
|----|---|----|----|-----|----|
| | 1 | 3 | -3 | -11 | -6 |
| 2 | | 2 | 10 | 14 | 6 |
| | 1 | 5 | 7 | 3 | 0 |
| -3 | | -3 | -6 | -3 | |
| | 1 | 2 | 1 | 0 | |
| -1 | | -1 | -1 | | |
| | 1 | 1 | 0 | | |

b) $x^5 + 6x^4 + 5x^3 - 24x^2 - 36x = x(x^4 + 6x^3 + 5x^2 - 24x - 36) = x(x-2)(x+2)(x+3)(x+3) =$

$= x(x-2)(x+2)(x+3)^2$

| | | | | | |
|----|---|----|-----|-----|-----|
| | 1 | 6 | 5 | -24 | -36 |
| 2 | | 2 | 16 | 42 | 36 |
| | 1 | 8 | 21 | 18 | 0 |
| -2 | | -2 | -12 | -18 | |
| | 1 | 6 | 9 | 0 | |
| -3 | | -3 | -9 | | |
| | 1 | 3 | 0 | | |

c) $x^4 + x^3 - 6x^2 - 4x + 8 = (x-2)(x-1)(x+2)(x+2) = (x-2)(x-1)(x+2)^2$

| | | | | | |
|----|---|----|----|----|----|
| | 1 | 1 | -6 | -4 | 8 |
| 2 | | 2 | 6 | 0 | -8 |
| | 1 | 3 | 0 | -4 | 0 |
| 1 | | 1 | 4 | 4 | |
| | 1 | 4 | 4 | 0 | |
| -2 | | -2 | -4 | | |
| | 1 | 2 | 0 | | |

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$$\begin{aligned} 3) \quad a) \quad & \frac{x+1}{2x-2} - \frac{x-1}{2x+2} - \frac{4x}{x^2-1} - \frac{x^2+1}{x^2-1} = \frac{x+1}{2(x-1)} - \frac{x-1}{2(x+1)} - \frac{4x}{(x+1)(x-1)} - \frac{x^2+1}{(x+1)(x-1)} = \\ & = \frac{(x+1)^2 - (x-1)^2 - 8x - 2(x^2+1)}{2(x-1)(x+1)} = \frac{(x^2+2x+1) - (x^2-2x+1) - 8x - 2x^2 - 2}{2(x+1)(x-1)} = \\ & = \frac{x^2+2x+1-x^2+2x-1-8x-2x^2-2}{2(x^2-1)} = \frac{-2x^2-4x-2}{2(x^2-1)} = \frac{-2(x+1)^2}{2(x+1)(x-1)} = \boxed{\frac{-(x+1)}{(x-1)}} \end{aligned}$$

$$b) \quad \frac{3x^2+1}{x^2+x} - \frac{2x}{x+1} = \frac{3x^2+1}{x(x+1)} - \frac{2x}{x+1} = \frac{3x^2+1-2x^2}{x(x+1)} = \boxed{\frac{x^2+1}{x^2+x}}$$

$$4) \quad a) \quad \sqrt{\frac{1}{9}} \cdot \sqrt[3]{27} = \frac{1}{3} \cdot 3 = \frac{3}{3} = \boxed{1}$$

$$b) \quad (\sqrt[4]{a})^{10} = ((a^{1/4})^{1/2})^{10} = (a^{1/8})^{10} = a^{10/8} = a^{5/4} = \sqrt[4]{a^5} = \boxed{a^4 \sqrt{a}}$$

$$c) \quad \sqrt{162a^5a^6} = \sqrt{2 \cdot 3^4 \cdot a^5 \cdot a^6} = 3^2 \cdot a^2 \cdot a^3 \sqrt{2a} = \boxed{9a^5 \sqrt{2a}}$$

$$d) \quad \sqrt[3]{32 \cdot x^4} = \sqrt[3]{2^5 \cdot x^4} = \sqrt[3]{2^3 \cdot 2^2 \cdot x^3 \cdot x} = \boxed{2x \sqrt[3]{4x}}$$

$$e) \quad \frac{\sqrt[4]{a^3b^5c}}{\sqrt{ab^3c^3}} = \frac{a^{3/4} \cdot b^{5/4} \cdot c^{1/4}}{a^{1/2} \cdot b^{3/2} \cdot c^{3/2}} = \frac{a^{1/4}}{b^{1/4} \cdot c^{5/4}} = \frac{\sqrt[4]{a}}{\sqrt[4]{bc^5}} = \boxed{\frac{\sqrt[4]{a}}{c^4 \sqrt[4]{bc}}}$$

$$f) \quad \sqrt[5]{64} = \sqrt[5]{2^6} = 2\sqrt[5]{2}$$

$$5. \quad a) \quad x^4 + 3x^3 - 3x^2 - 11x - 6 = 0$$

Del ejercicio 2a

$$(x-2)(x+3)(x+1)^2 = 0$$

soluciones:

$$\begin{aligned} x_1 &= 2 \\ x_2 &= -3 \\ x_3 &= -1 \text{ (doble)} \end{aligned}$$

$$b) \quad x^4 - 13x^2 + 36 = 0$$

$$x^2 = t \quad t^2 - 13t + 36 = 0$$

$$t = \frac{13 \pm \sqrt{169 - 144}}{2} = \frac{13 \pm 5}{2} = \begin{matrix} 9 \\ 4 \end{matrix}$$

$$x = \pm \sqrt{t}$$

$$\begin{aligned} x_1 &= 3 \\ x_2 &= -3 \\ x_3 &= -2 \\ x_4 &= 2 \end{aligned}$$

$$c) 5x^2 - 7x + 3 = 0$$

$$x = \frac{7 \pm \sqrt{49 - 60}}{10} = \frac{7 \pm \sqrt{-11}}{10} = \boxed{\text{no solución en } \mathbb{R}}$$

$$d) x^4 + 5x^2 + 4 = 0$$

$$x^2 = t \quad t^2 + 5t + 4 = 0$$

$$t = \frac{-5 \pm \sqrt{25 - 16}}{2} = \frac{-5 \pm 3}{2} = \begin{cases} -1 \\ -4 \end{cases}$$

$$x = \pm \sqrt{t}$$

$$\left[\begin{array}{l} x_1 = +\sqrt{-1} \Rightarrow \text{no solución real} \\ x_2 = -\sqrt{-1} \Rightarrow \text{no solución real} \\ x_3 = +\sqrt{-4} \Rightarrow \text{no solución real} \\ x_4 = -\sqrt{-4} \Rightarrow \text{no solución real} \end{array} \right]$$

$$e) \frac{x+1}{x^2-2x} + \frac{x-1}{x} = 2$$

$$\frac{x+1}{x(x-2)} + \frac{x-1}{x} = 2$$

$$\frac{x+1+(x-1)(x-2)}{x(x-2)} = 2$$

$$x+1+x^2-2x-x+2 = 2x(x-2)$$

$$x^2 - 2x + 3 = 2x^2 - 4x$$

$$x^2 - 2x + 3 - 2x^2 + 4x = 0$$

$$-x^2 + 2x + 3 = 0$$

$$x = \frac{-2 \pm \sqrt{4 + 12}}{-2} = \frac{-2 \pm 4}{-2} = \begin{cases} -1 \\ 3 \end{cases}$$

$$\text{solución } \boxed{\begin{array}{l} x_1 = -1 \\ x_2 = 3 \end{array}}$$

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6.

$$a) \begin{cases} 5x + y = 8 \Rightarrow y = 8 - 5x \\ 3x - y = 11 \end{cases}$$

$$8x = 19$$

$$\boxed{x = \frac{19}{8}} \rightarrow \boxed{y = 8 - \frac{95}{8} = \frac{64 - 95}{8} = \frac{-31}{8}}$$

$$b) \begin{cases} 3x - 5y = -26 \rightarrow 6x - 10y = -52 \\ 4x + 10y = 32 \end{cases}$$

$$10x = -20$$

$$\boxed{y = \frac{26 + 3x}{5} = \frac{26 - 6}{5} = \frac{20}{5} = 4} \quad \boxed{x = \frac{-20}{10} = -2}$$

$$c) \begin{cases} y + 8 = x^2 \leftarrow \\ y - 2x = 0 \rightarrow y = 2x \end{cases}$$

$$2x + 8 = x^2$$

$$x^2 - 2x - 8 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 32}}{2} = \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2} \rightarrow \begin{matrix} 4 \\ -2 \end{matrix}$$

$$d) \begin{cases} x_1 = 4 \rightarrow y_1 = 8 \\ x_2 = -2 \rightarrow y_2 = -4 \end{cases}$$

$$d) \begin{cases} x - y = 15 \rightarrow x = 15 + y \\ xy = 100 \leftarrow \end{cases}$$

$$(15 + y) \cdot y = 100$$

$$15y + y^2 = 100$$

$$y^2 + 15y - 100 = 0$$

$$y = \frac{-15 \pm \sqrt{225 + 400}}{2} = \frac{-15 \pm 25}{2} \rightarrow \begin{matrix} 5 \\ -20 \end{matrix}$$

$$\begin{cases} y_1 = 5 \rightarrow x_1 = 20 \\ y_2 = -20 \rightarrow x_2 = -5 \end{cases}$$

$$\textcircled{e} \begin{cases} x^2 + y^2 = 58 \\ x^2 - y^2 = 40 \end{cases} \rightarrow \begin{aligned} y^2 &= 58 - x^2 \\ y &= \pm \sqrt{58 - x^2} \end{aligned}$$

$$2x^2 = 98$$

$$x^2 = 49$$

$$x = \pm 7$$

$$\begin{array}{l} \text{S: } x_1 = 7 \rightarrow y_1 = +3 \\ x_2 = -7 \rightarrow y_2 = -3 \\ x_3 = 7 \rightarrow y_3 = -3 \\ x_4 = -7 \rightarrow y_4 = 3 \end{array}$$

$$\begin{aligned} \textcircled{7} \text{ a) } \sqrt{48} - \sqrt{12} + \sqrt{3} &= \sqrt{2^4 \cdot 3} - \sqrt{2^2 \cdot 3} + \sqrt{3} = 4\sqrt{3} - 2\sqrt{3} + \sqrt{3} = \boxed{3\sqrt{3}} \\ \text{b) } \sqrt{28} - \sqrt{7} + \sqrt{63} &= \sqrt{2^2 \cdot 7} - \sqrt{7} + \sqrt{3^2 \cdot 7} = 2\sqrt{7} - \sqrt{7} + 3\sqrt{7} = \boxed{4\sqrt{7}} \\ \text{c) } \sqrt[3]{54} + \sqrt[3]{2} &= \sqrt[3]{3^3 \cdot 2} + \sqrt[3]{2} = 3\sqrt[3]{2} + \sqrt[3]{2} = \boxed{4\sqrt[3]{2}} \\ \text{d) } \sqrt[3]{81} - \sqrt[3]{24} &= \sqrt[3]{3^4} - \sqrt[3]{2^3 \cdot 3} = 3\sqrt[3]{3} - 2\sqrt[3]{3} = \boxed{\sqrt[3]{3}} \end{aligned}$$

$$\begin{aligned} \textcircled{8} \text{ a) } \frac{3^{-4} \cdot 5^3 \cdot 2}{3^3 \cdot 5^{-4} \cdot 2^{-2}} &= \frac{5^3 \cdot 2 \cdot 5^4 \cdot 2^2}{3^3 \cdot 3^4} = \frac{5^7 \cdot 2^3}{3^7} = \boxed{\frac{625000}{2187}} \\ \text{b) } \frac{2^7 \cdot 2^{-6} \cdot 2^3}{2^{-5}} &= \frac{2^7 \cdot 2^3 \cdot 2^5}{2^6} = \frac{2^{15}}{2^6} = 2^9 = \boxed{512} \end{aligned}$$